A long tungsten-core solenoid carries a current. (a) If the core is removed while the current is held constant, does the magnetic field strength in the region inside the solenoid decrease or increase? (b) By what percentage does the magnetic field strength in the region inside the solenoid decrease or increase?

**Picture the Problem** We can use \( B = B_{\text{app}}(1 + \chi_m) \) to relate \( B \) and \( B_{\text{app}} \) to the magnetic susceptibility of tungsten. Dividing both sides of this equation by \( B_{\text{app}} \) and examining the value of \( \chi_m \text{, tungsten} \) will allow us to decide whether the field inside the solenoid decreases or increases when the core is removed.

Express the magnetic field inside the solenoid with the tungsten core present \( B \) in terms of \( B_{\text{app}} \) and \( \chi_m \):

\[
B = B_{\text{app}}(1 + \chi_m)
\]

where \( B_{\text{app}} \) is the magnetic field in the absence of the tungsten core.

Express the ratio of \( B \) to \( B_{\text{app}} \):

\[
\frac{B}{B_{\text{app}}} = 1 + \chi_m \tag{1}
\]

(a) Because \( \chi_{m \text{, tungsten}} > 0 \):

\( B > B_{\text{app}} \) and \( B \) will decrease when the tungsten core is removed.

(b) From equation (1), the fractional change is:

\[
\chi_m = 6.8 \times 10^{-5} = \frac{6.8 \times 10^{-5}}{100} \%
\]

Nickel has a density of 8.70 g/cm\(^3\) and a molar mass of 58.7 g/mol. Nickel’s saturation magnetization is 0.610 T. Calculate the magnetic moment of a nickel atom in Bohr magnetons.

**Picture the Problem** We can find the magnetic moment of a nickel atom \( \mu \) from its relationship to the saturation magnetization \( M_S \) using \( M_S = n\mu \) where \( n \) is the number of molecules per unit volume. \( n \), in turn, can be found from Avogadro’s number, the density of nickel, and its molar mass using \( n = \frac{N_A \rho}{M} \).

Express the saturation magnetic field in terms of the number of molecules

\[
M_S = n\mu \Rightarrow \mu = \frac{M_S}{n}
\]
per unit volume and the magnetic moment of each molecule:

Express the number of molecules per unit volume in terms of Avogadro’s number \( N_A \), the molecular mass \( M \), and the density \( \rho \):

Substitute for \( n \) in the equation for \( \mu \) and simplify to obtain:

Substitute numerical values and evaluate \( \mu \):

Divide \( \mu \) by \( \mu_0 = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2 \) to obtain:

A long thin solenoid has 50 turns/cm and carries a current of 2.00 A. The solenoid is filled with iron and the magnetic field is measured to be 1.72 T. (a) Neglecting end effects, what is the magnitude of the applied magnetic field? (b) What is the magnetization? (c) What is the relative permeability?

### Picture the Problem
We can use the equation describing the magnetic field on the axis of a solenoid, as a function of the current in the solenoid, to find \( B_{\text{app}} \). We can then use \( B = B_{\text{app}} + \mu_0 M \) to find \( M \) and \( B = K_m B_{\text{app}} \) to evaluate \( K_m \).

(a) Relate the magnetic field on the axis of a solenoid to the current in the solenoid:

Substitute numerical values and evaluate \( B_{\text{app}} \):

(b) Relate \( M \) to \( B \) and \( B_{\text{app}} \):

\[ B = B_{\text{app}} + \mu_0 M \Rightarrow M = \frac{B - B_{\text{app}}}{\mu_0} \]
Substitute numerical values and evaluate $M$:

$$M = \frac{1.72 \, \text{T} - 12.6 \, \text{mT}}{4\pi \times 10^{-7} \, \text{N/A}^2}$$

$$= \frac{1.36 \times 10^6}{4\pi \times 10^{-7}} \text{A/m}$$

(c) Express $B$ in terms of $K_m$ and $B_{app}$:

$$B = K_m B_{app} \Rightarrow K_m = \frac{B}{B_{app}}$$

Substitute numerical values and evaluate $K_m$:

$$K_m = \frac{1.72 \, \text{T}}{12.6 \, \text{mT}} = \frac{137}{10}$$

91. An infinitely long straight wire is bent, as shown in Figure 27-66. The circular portion has a radius of 10.0 cm and its center a distance $r$ from the straight part. Find $r$ so that the magnetic field at the region occupied by the center of the circular portion is zero.

**Picture the Problem** Let the positive $x$ direction be out of the page. We can use the expressions for the magnetic fields due to an infinite straight line and a circular loop to express the net magnetic field at the center of the circular loop. We can set this net field to zero and solve for $r$.

Express the net magnetic field at the center of circular loop:

$$\vec{B} = \vec{B}_{loop} + \vec{B}_{line}$$

Letting $R$ represent the radius of the loop, express $\vec{B}_{loop}$:

$$\vec{B}_{loop} = -\frac{\mu_0 I}{2R} \hat{i}$$

Express the magnetic field due to the current in the infinite straight line:

$$\vec{B}_{line} = \frac{\mu_0 I}{2\pi r} \hat{i}$$

Substitute for $\vec{B}_{loop}$ and $\vec{B}_{line}$ in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0 I}{2R} \hat{i} + \frac{\mu_0 I}{2\pi r} \hat{i} = \left(-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r}\right) \hat{i}$$

If $\vec{B} = 0$, then:

$$-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r} = 0 \Rightarrow -\frac{1}{R} + \frac{1}{\pi r} = 0$$

Solving for $r$ yields:

$$r = \frac{10.0 \, \text{cm}}{\pi} = 3.18 \, \text{cm}$$
Chapter 28

6  •  Give the direction of the induced current in the circuit, shown on the right in Figure 28-37, when the resistance in the circuit on the left is suddenly 
(a) increased and (b) decreased. Explain your answer.

Determine the Concept  The induced emf and induced current in the circuit on the right are in such a direction as to oppose the change that produces them (Lenz’s Law). We can determine the direction of the induced current in the circuit. Note that when $R$ is constant, $\vec{B}$ in the circuit to the right points out of the paper.

(a) If $R$ increases, $I$ decreases and $B$ in the circuit to the right decreases. Lenz’s law tells us that the induced current is counterclockwise.

(b) If $R$ decreases, $I$ increases and $B$ in the circuit to the right increases. Lenz’s law tells us that the induced current is clockwise.

8  •  A bar magnet moves with constant velocity along the axis of a loop, as shown in Figure 28-39, (a) Make a graph of the magnetic flux through the loop as a function of time. Indicate on the graph when the magnet is halfway through the loop by designating this time $t_1$. Choose the direction of the normal to the flat surface bounded by the flat surface to be to the right. (b) Make a graph of the induced current in the loop as a function of time. Choose the positive direction for the current to be clockwise as viewed from the left.

Determine the Concept  We know that, as the magnet moves to the right, the flux through the loop first increases until the magnet is halfway through the loop and then decreases. Because the flux first increases and then decreases, the current will change directions, having its maximum values when the flux is changing most rapidly.

(a) and (b) The following graph shows the flux and the induced current as a function of time as the bar magnet passes through the coil. When the center of the magnet passes through the plane of the coil $d\phi_m/dt = 0$ and the current is zero.
28  ⋯ (a) Compute the magnetic flux through the rectangular loop shown in Figure 28-45. (b) Evaluate your answer for \( a = 5.0 \, \text{cm} \), \( b = 10 \, \text{cm} \), \( d = 2.0 \, \text{cm} \), and \( I = 20 \, \text{A} \).

**Picture the Problem** We can use the hint to set up the element of area \( dA \) and express the flux \( d\phi_m \) through it and then carry out the details of the integration to express \( \phi_m \).

(a) The flux through the strip of area \( dA \) is given by:

\[
d\phi_m = B dA
\]

where \( dA = bdx \).

Express \( B \) at a distance \( x \) from a long, straight wire:

\[
B = \frac{\mu_0}{4\pi} \frac{2I}{x} = \frac{\mu_0}{2\pi} \frac{I}{x}
\]

Substitute to obtain:

\[
d\phi_m = \frac{\mu_0}{2\pi} \frac{I}{x} bdx = \frac{\mu_0 I b}{2\pi} \frac{dx}{x}
\]

Integrate from \( x = d \) to \( x = d + a \):

\[
\phi_m = \frac{\mu_0 I b}{2\pi} \int_{d}^{d+a} \frac{dx}{x} = \frac{\mu_0 I b}{2\pi} \ln \left( \frac{d + a}{d} \right)
\]

(b) Substitute numerical values and evaluate \( \phi_m \):

\[
\phi_m = \left( 4\pi \times 10^{-7} \, \text{N/A}^2 \right) (20 \, \text{A})(0.10 \, \text{m}) \ln \left( \frac{7.0 \, \text{cm}}{2.0 \, \text{cm}} \right) = 0.50 \, \mu\text{Wb}
\]
At the equator, a 1000-turn coil that has a cross-sectional area of 300 cm$^2$ and a resistance of 15.0 $\Omega$ is aligned so that its plane is perpendicular to Earth’s magnetic field of 0.700 G. (a) If the coil is flipped over in 0.350 s, what is the average induced current in it during the 0.350 s? (b) How much charge flows through a cross section of the coil wire during the 0.350 s?

**Picture the Problem** (a) Because the average induced current is proportional to the induced emf and the induced emf, in turn, is given by Faraday’s law, we can find $I_{av}$ from the change in the magnetic flux through the coil, the resistance of the coil, and the time required for the flipping of the coil. (b) Knowing the average current, we can use its definition to find the charge flowing in the coil.

(a) The average induced current is given by:

$$I_{av} = \frac{E}{R}$$

The induced emf in the coil is the rate at which the magnetic flux is changing:

$$E = \frac{\Delta \phi_{m}}{\Delta t} = \frac{2 \phi_{m}}{\Delta t} = \frac{2NBA}{\Delta t}$$

Substituting for $E$ yields:

$$I_{av} = \frac{2NBA}{R\Delta t}$$

Substitute numerical values and evaluate $I_{av}$:

$$I_{av} = \frac{2(1000) \left( 0.700 \, G \times \frac{1 \, T}{10^4 \, G} \right) \left( 300 \, cm^2 \times \left( \frac{1 \, m}{10^2 \, cm} \right)^2 \right)}{(15.0 \, \Omega)(0.350 \, s)} = 800 \, \mu A$$

(b) The average current is also given by:

$$I_{av} = \frac{\Delta Q}{\Delta t} \Rightarrow \Delta Q = I_{av} \Delta t$$

Substitute numerical values and evaluate $\Delta Q$:

$$\Delta Q = (0.800 \, mA)(0.350 \, s) = 280 \, \mu C$$