Homework #1
Chapter 21
The Electric Field I: Discrete Charge Distributions

2  •  A carbon atom can become a carbon ion if it has one or more of its electrons removed during a process called ionization. What is the net charge on a carbon atom that has had two of its electrons removed? (a) +e, (b) –e, (c) +2e, (d) –2e

Determine the Concept If two electrons are removed from a carbon atom, it will have a net positive charge of +2e. (c) is correct.

6  ••  A metal sphere is positively charged. Is it possible for the sphere to electrically attract another positively charged ball? Explain your answer.

Determine the Concept Yes. Because a metal sphere is a conductor, the proximity of a positively charged ball (not necessarily a conductor), will induce a redistribution of charges on the metal sphere with the surface nearer the positively charged ball becoming negatively charged. Because the negative charges on the metal sphere are closer to the positively charged ball than are the positive charges on the metal sphere, the net force will be attractive.

9  ••  Two neutral conducting spheres are in contact and are supported on a large wooden table by insulated stands. A positively charged rod is brought up close to the surface of one of the spheres on the side opposite its point of contact with the other sphere. (a) Describe the induced charges on the two conducting spheres, and sketch the charge distributions on them. (b) The two spheres are separated and then the charged rod is removed. The spheres are then separated far apart. Sketch the charge distributions on the separated spheres.

Determine the Concept Because the spheres are conductors, there are free electrons on them that will reposition themselves when the positively charged rod is brought nearby.

(a) On the sphere near the positively charged rod, the induced charge is negative and near the rod. On the other sphere, the net charge is positive and on the side far from the rod. This is shown in the diagram.
(b) When the spheres are separated and far apart and the rod has been removed, the induced charges are distributed uniformly over each sphere. The charge distributions are shown in the diagram.

18  Estimate the force required to bind the two protons in the He nucleus together. HINT: Model the protons as point charges. You will need to have an estimate of the distance between them.

Picture the Problem Because the nucleus is in equilibrium, the binding force must be equal to the electrostatic force of repulsion between the protons.

Apply \( \sum \vec{F} = 0 \) to a proton to obtain: \( F_{\text{binding}} - F_{\text{electrostatic}} = 0 \)

Solve for \( F_{\text{binding}} \): \( F_{\text{binding}} = F_{\text{electrostatic}} \)

Using Coulomb’s law, substitute for \( F_{\text{electrostatic}} \): \( F_{\text{binding}} = \frac{kq^2}{r^2} \)

Assuming the diameter of the He nucleus to be approximately \( 10^{-15} \) m, substitute numerical values and evaluate \( F_{\text{electrostatic}} \):

\[
F_{\text{binding}} = \frac{8.988 \times 10^9 \text{ Nm}^2/\text{C}^2 \left(1.602 \times 10^{-19} \text{ C}\right)^2}{\left(10^{-15} \text{ m}\right)^2} \approx 0.2 \text{ kN}
\]

25  During a process described by the photoelectric effect, ultra-violet light can be used to charge a piece of metal. (a) If such light is incident on a slab of conducting material and electrons are ejected with enough energy that they escape the surface of the metal, how long before the metal has a net charge of +1.50 nC if 1.00 \( \times \) \( 10^6 \) electrons are ejected per second? (b) If 1.3 eV is needed to eject an electron from the surface, what is the power rating of the light beam? (Assume this process is 100% efficient.)

Picture the Problem (a) The required time is the ratio of the charge that accumulates to the rate at which it is delivered to the conductor. (b) We can use the definition of power to find the power rating of the light beam.

(a) The required time is the ratio of the charge that accumulates to the rate at which it is delivered: \( \Delta t = \frac{\Delta q}{I} = \frac{\Delta q}{dq/dt} \)
The Electric Field I: Discrete Charge Distributions

Substitute numerical values and evaluate $\Delta t$:

$$\Delta t = \frac{1.50 \text{ nC}}{1.60 \times 10^{-10} \text{ C/electron}} \left( \frac{1.00 \times 10^6 \text{ electrons/s}}{1.602 \times 10^{-19} \text{ C/electron}} \right) = 9.36 \times 10^3 \text{ s} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

$$= 2.60 \text{ h}$$

(b) The power rating of the light beam is the rate at which it delivers energy:

The energy delivered by the beam is the product of the energy per electron, the electron current (that is, the number of electrons removed per unit time), and the elapsed time:

Substituting for $\Delta E$ in the expression for $P$ and simplifying yields:

$$P = \frac{\Delta E}{\Delta t}$$

$$\Delta E = E_{\text{per electron}} I_{\text{electron}} \Delta t$$

Substitute numerical values and evaluate $P$:

$$P = \left( 1.3 \frac{\text{ eV}}{\text{ electron}} \times \frac{1.60 \times 10^{-19} \text{ J}}{\text{ eV}} \right) \left( \frac{1.00 \times 10^6 \text{ electrons/s}}{1.00 \times 10^6 \text{ electrons/s}} \right) = 2.1 \times 10^{-13} \text{ W}$$

36  The structure of the NH$_3$ molecule is approximately that of an equilateral tetrahedron, with three H$^+$ ions forming the base and an N$^{3-}$ ion at the apex of the tetrahedron. The length of each side is $1.64 \times 10^{-10}$ m. Calculate the electric force that acts on each ion.

**Picture the Problem** Let the H$^+$ ions be in the x-y plane with H$_1$ at (0, 0, 0), H$_2$ at $(a, 0, 0)$, and H$_3$ at $\left( \frac{a}{2}, \frac{a\sqrt{3}}{2}, 0 \right)$. The N$^{3-}$ ion, with charge $q_4$ in our notation, is then at $\left( \frac{a}{2}, \frac{a}{2\sqrt{3}}, \frac{a}{3} \right)$ where $a = 1.64 \times 10^{-10}$ m. To simplify our calculations we’ll set $ke^2/a^2 = C = 8.56 \times 10^{-9}$ N. We can apply Coulomb’s law and the principle of superposition of forces to find the net force acting on each ion.
Express the net force acting on point charge \( q_1 \):

\[
\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}
\]

Find \( \vec{F}_{2,1} \):

\[
\vec{F}_{2,1} = \frac{kq_1q_2}{r_{2,1}^2} \hat{r}_{2,1} = C\left(-\hat{i}\right) = -C\hat{i}
\]

Find \( \vec{F}_{3,1} \):

\[
\vec{F}_{3,1} = \frac{kq_1q_3}{r_{3,1}^2} \hat{r}_{3,1} = \frac{C}{a} \left(0 - \frac{a}{2}\right)\hat{i} + \left(0 - \frac{a\sqrt{3}}{2}\right)\hat{j}
\]

\[
= -C\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)
\]

Noting that the magnitude of point charge \( q_4 \) is three times that of the other point charges and that it is negative, express \( \vec{F}_{4,1} \):
\[ \vec{F}_{4,1} = 3C \vec{r}_{4,1} = -3C \frac{\left( 0 - \frac{a}{2} \right) \hat{i} + \left( 0 - \frac{a}{2\sqrt{3}} \right) \hat{j} + \left( 0 - \frac{a\sqrt{2}}{\sqrt{3}} \right) \hat{k}}{\sqrt{\left( \frac{a}{2} \right)^2 + \left( \frac{a}{2\sqrt{3}} \right)^2 + \left( \frac{a\sqrt{2}}{\sqrt{3}} \right)^2}} \]

\[ = 3C \frac{\left( \frac{a}{2} \right) \hat{i} + \left( \frac{a}{2\sqrt{3}} \right) \hat{j} + \left( \frac{a\sqrt{2}}{\sqrt{3}} \right) \hat{k}}{a} = 3C \left[ \left( \frac{1}{2} \right) \hat{i} + \left( \frac{1}{2\sqrt{3}} \right) \hat{j} + \left( \frac{\sqrt{2}}{\sqrt{3}} \right) \hat{k} \right] \]

Substitute in the expression for \( \vec{F}_1 \) to obtain:

\[ \vec{F}_1 = -C \hat{\vec{i}} - C \left( \frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j} \right) + 3C \left[ \left( \frac{1}{2} \right) \hat{i} + \left( \frac{1}{2\sqrt{3}} \right) \hat{j} + \left( \frac{\sqrt{2}}{\sqrt{3}} \right) \hat{k} \right] = C\sqrt{6}\hat{k} \]

From symmetry considerations:

\[ \vec{F}_2 = \vec{F}_3 = \vec{F}_1 = C\sqrt{6}\hat{k} \]

Express the condition that the molecule is in equilibrium:

\[ \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0 \]

Solve for and evaluate \( \vec{F}_4 \):

\[ \vec{F}_4 = -\left( \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \right) = -3\vec{F}_1 = -3C\sqrt{6}\hat{k} \]

The electric field near the surface of Earth points downward and has a magnitude of 150 N/C. (a) Compare the magnitude of the upward electric force on an electron with the magnitude of the gravitational force on the electron. (b) What charge should be placed on a ping pong ball of mass 2.70 g so that the electric force balances the weight of the ball near Earth’s surface?

**Picture the Problem** We can compare the electric and gravitational forces acting on an electron by expressing their ratio. Because the ping pong ball is in equilibrium under the influence of the electric and gravitational forces acting on it, we can use the condition for translational equilibrium to find the charge that would have to be placed on it in order to balance Earth’s gravitational force on it.

(a) Express the magnitude of the electric force acting on the electron:

\[ F_e = eE \]
Express the magnitude of the gravitational force acting on the electron:

\[ F_g = m_e g \]

Express the ratio of these forces to obtain:

\[ \frac{F_e}{F_g} = \frac{eE}{mg} \]

Substitute numerical values and evaluate \( F_e/F_g \):

\[ \frac{F_e}{F_g} = \frac{(1.602 \times 10^{-19} \text{ C})(150 \text{ N/C})}{(9.109 \times 10^{-31} \text{ kg})(9.81 \text{ m/s}^2)} = 2.69 \times 10^{12} \]

or

\[ F_e = (2.69 \times 10^{12})F_g \]

Thus, the electric force is greater by a factor of \( 2.69 \times 10^{12} \).

(b) Letting the upward direction be positive, apply the condition for static equilibrium to the ping pong ball to obtain:

\[ F_e - F_g = 0 \]

or

\[-qE - mg = 0 \Rightarrow q = -\frac{mg}{E} \]

Substitute numerical values and evaluate \( q \):

\[ q = -\frac{(3.00 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2)}{150 \text{ N/C}} = -0.196 \text{ mC} \]

56 ** A charged particle leaves the origin with a speed of \( 3.00 \times 10^6 \text{ m/s} \) at an angle of \( 35^\circ \) above the \( x \) axis. A uniform electric field given by \( \vec{E} = -E_0 \hat{j} \) exists throughout the region. Find \( E_0 \) such that the particle will cross the \( x \) axis at \( x = 1.50 \text{ cm} \) if the particle is (a) an electron, and (b) a proton.

**Picture the Problem** We can use constant-acceleration equations to express the \( x \) and \( y \) coordinates of the particle in terms of the parameter \( t \) and Newton’s 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for \( y \) as a function of \( x \), \( q \), and \( m \) that we can solve for \( E_y \).

Express the \( x \) and \( y \) coordinates of the particle as functions of time:

\[ x = (v \cos \theta) t \]

and

\[ y = (v \sin \theta) t - \frac{1}{2} a_y t^2 \]
Apply Newton’s 2nd law to relate the acceleration of the particle to the net force acting on it:

\[ a_y = \frac{F_{net,y}}{m} = \frac{qE_y}{m} \]

Substitute in the y-coordinate equation to obtain:

\[ y = (v \sin \theta) t - \frac{qE_y}{2m} t^2 \]

Eliminate the parameter \( t \) between the two equations to obtain:

\[ y = (\tan \theta)x - \frac{qE_y}{2mv^2 \cos^2 \theta} x^2 \]

Set \( y = 0 \) and solve for \( E_y \) to obtain:

\[ E_y = \frac{mv^2 \sin 2\theta}{q} \]

Substitute the non-particle-specific data to obtain:

\[ E_y = \frac{m(3.00 \times 10^6 \text{ m/s})^2 \sin 70^\circ}{q(0.0150 \text{ m})} \]

\[ = \left(5.638 \times 10^{14} \text{ m/s}^2\right) \frac{m}{q} \]

\( a \) Substitute for the mass and charge of an electron and evaluate \( E_y \):

\[ E_y = \left(5.638 \times 10^{14} \text{ m/s}^2\right) \frac{9.109 \times 10^{-31} \text{ kg}}{1.602 \times 10^{-19} \text{ C}} \]

\[ = -3.2 \text{ kN/C} \]

\( b \) Substitute for the mass and charge of a proton and evaluate \( E_y \):

\[ E_y = \left(5.64 \times 10^{14} \text{ m/s}^2\right) \frac{1.673 \times 10^{-27} \text{ kg}}{1.602 \times 10^{-19} \text{ C}} \]

\[ = 5.9 \text{ MN/C} \]

Two point charges, \( q_1 = 2.0 \text{ pC} \) and \( q_2 = -2.0 \text{ pC} \), are separated by 4.0 \( \mu \text{m} \). \( a \) What is the magnitude of the dipole moment of this pair of charges? \( b \) Sketch the pair and show the direction of the dipole moment.

**Picture the Problem** We can use its definition to find the dipole moment of this pair of charges.

\( a \) Apply the definition of electric dipole moment to obtain:

\[ \vec{p} = q\vec{L} \]

and

\[ p = (2.0 \text{ pC})(4.0 \mu\text{m}) \]

\[ = 8.0 \times 10^{-18} \text{ C} \cdot \text{m} \]

\( b \) If we assume that the dipole is
oriented as shown to the right, then \( \vec{p} \) is to the right; pointing from the negative charge toward the positive charge.

83  

Two equal positive point charges \( Q \) are on the \( x \) axis at \( x = \frac{1}{2} a \) and \( x = -\frac{1}{2} a \). (a) Obtain an expression for the electric field on the \( y \) axis as a function of \( y \). (b) A bead of mass \( M \), which has a charge \( q \), moves along the \( y \) axis on a thin frictionless taut thread. Find the electric force that acts on the bead as a function of \( y \) and determine the sign of \( q \) such that this force always points away from the origin. (c) The bead is initially at rest at the origin. If it is given a slight nudge in the \( +y \) direction, how fast will the bead be traveling the instant the net force on it is a maximum? (Assume any effects due to gravity are negligible.)

**Picture the Problem**  
(a) We can use Coulomb’s law for the electric field due to a point charge and superposition of fields to find the electric field at any point on the \( y \) axis. (b) Using the definition of electric field will yield an expression for the electric force that acts on the bead. In Part (c) we can set \( dF_y / dy = 0 \) to find the value of \( y \) that maximizes \( F_y \) and then use the work-kinetic energy theorem (integrating \( F_y \) from 0 to this extreme value will yield an expression for the work done on the bead during this displacement) to find the speed of the bead when the force acting on it is a maximum.

\( a \)

Use Coulomb’s law for the electric field due to a point charge and superposition of fields, to express the field at point \( P \) on the \( y \) axis:

\[
\vec{E}_y = \vec{E}_1 + \vec{E}_2
\]

\[
= \frac{kq_1}{r_{1,p}^2} \hat{r}_{1,p} + \frac{kq_2}{r_{2,p}^2} \hat{r}_{2,p}
\]

The unit vectors \( \hat{r}_{1,p} \) and \( \hat{r}_{2,p} \) are given by:

\[
\hat{r}_{1,p} = \frac{\hat{r}_{1,p}}{r_{1,p}} = -\frac{1}{2} \hat{a} + \frac{y}{r_{1,p}} \hat{j}
\]

and

\[
\hat{r}_{2,p} = \frac{\hat{r}_{2,p}}{r_{2,p}} = \frac{1}{2} \hat{a} + \frac{y}{r_{2,p}} \hat{j}
\]
Substituting for \( q_1, q_2, \hat{r}_{1,p}, \hat{r}_{2,p} \) and \( r_{1,p} = r_{2,p} = \left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{1/2} \) yields:

\[
\hat{E}_y = \frac{kQ}{r_{1,p}^2} \left( -\frac{1}{2} \hat{a} + \hat{j} \right) + \frac{kQ}{r_{2,p}^2} \left( \frac{1}{2} \hat{a} + \hat{j} \right) = \frac{kQ}{r_{1,p}^3} \left( -\frac{1}{2} \hat{a} + \hat{j} \right) + \frac{kQ}{r_{2,p}^3} \left( \frac{1}{2} \hat{a} + \hat{j} \right)
\]

\[
= \frac{kQ}{\left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{3/2}} \left( \frac{kQ}{\left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{3/2}} \right) (\hat{j}) = \frac{2kQy}{\left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{3/2}} \hat{j}
\]

\[(b) \text{ Relate the electric force on the bead to its charge and the electric field:}\]

\[
\hat{F} = q\hat{E}_y = \frac{2kQy}{\left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{3/2}} \hat{j}
\]

where \( q \) must be positive if \( \hat{F} \) always points away from the origin.

\[(c) \text{ Apply the work-kinetic energy theorem to the bead as it moves from the origin to the location at which } F_y \text{ is a maximum to obtain:}\]

\[
W_{\text{net}} = \Delta K = K_f - K_i
\]

or, because \( K_i = 0 \),

\[
W_{\text{done by } F_y} = \frac{1}{2} M v_f^2
\]

Solving for \( v_f \) yields:

\[
v_f = \sqrt{\frac{2W_{\text{done by } F_y}}{M}}
\]

The work done on the bead by \( F_y \) as the bead moves from the origin to \( y_{\text{max}} \) (the \( y \) coordinate of the point at which \( F_y \) is a maximum) is given by:

\[
W_{\text{done by } F_y} = \int_0^{y_{\text{max}}} \frac{2kQy}{\left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{3/2}} dy
\]

Substituting for \( W_{\text{done by } F_y} \) and simplifying yields:

\[
v_f = \sqrt{\frac{4kQy_{\text{max}}}{m} \int_0^{y_{\text{max}}} \left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{3/2} dy}
\]

The condition that \( F_y \) is a maximum is:

\[
\frac{dF_y}{dy} = \frac{d}{dy} \left[ \frac{y}{\left[ y^2 + \left( \frac{1}{2} a \right)^2 \right]^{3/2}} \right] = 0
\]

Carrying out the details of the differentiation and solving for the critical value \( y_{\text{max}} \) yields:

\[
y_{\text{max}} = \frac{a}{2\sqrt{2}} \text{ where we've ignored the negative value because the bead is given a nudge in the } +y \text{ direction.}
\]
Chapter 21

Substitute for \( y_{\text{max}} \) in the expression for \( v_f \) to obtain:

\[
v_f = \sqrt{\frac{4kqQ}{M} \int_{0}^{a} \frac{y}{y^2 + \left(\frac{y}{a}\right)^2} \, dy}
\]

Evaluating the integral yields:

\[
\int_{0}^{a} \frac{y}{y^2 + \left(\frac{y}{a}\right)^2} \, dy \approx \frac{0.367}{a}
\]

Substitute for \( \int_{0}^{a} \frac{y}{y^2 + \left(\frac{y}{a}\right)^2} \, dy \) and simplify to obtain:

\[
v_f = \sqrt{\frac{4kqQ \left( \frac{0.367}{a} \right)}{M}} = \frac{kqQ}{aM}
\]