24.52. Cell Membranes. Cell membranes (the walled enclosure around a cell) are typically about 7.5 nm thick. They are partially permeable to allow charged material to pass in and out, as needed. Equal but opposite charge densities build up on the inside and outside faces of such a membrane, and these charges prevent additional charges from passing through the cell wall. We can model a cell membrane as a parallel-plate capacitor, with the membrane itself containing proteins embedded in an organic material to give the membrane a dielectric constant of about 10. (See Fig. 24.30.) (a) What is the capacitance per square centimeter of such a cell wall? (b) In its normal resting state, a cell has a potential difference of 85 mV across its membrane. What is the electric field inside this membrane?

\[ C = \varepsilon_0 \frac{A}{d}, \quad V = Ed \text{ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.} \]

**Set Up:**  \[ A = 1.0 \text{ cm}^2 = 1.0 \times 10^{-4} \text{ m}^2. \]

**Execute:**

(a) \[ C = \left(10 \right) \frac{(8.85 \times 10^{-12} \text{ F/m})(1.0 \times 10^{-4} \text{ m}^3)}{7.5 \times 10^{-9} \text{ m}} = 1.18 \mu \text{F per cm}^2. \]

(b) \[ E = \frac{V}{d} = \frac{85 \text{ mV}}{7.5 \times 10^{-9} \text{ m}} = 1.13 \times 10^7 \text{ V/m}. \]

**Evaluate:** The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

24.66. An air capacitor is made by using two flat plates, each with area \( A \), separated by a distance \( d \). Then a metal slab having thickness \( a \) (less than \( d \)) and the same shape and size as the plates is inserted between them, parallel to the plates and not touching either plate (Fig. 24.36). (a) What is the capacitance of this arrangement? (b) Express the capacitance as a multiple of the capacitance \( C_0 \) when the metal slab is not present. (c) Discuss what happens to the capacitance in the limits \( a \to 0 \) and \( a \to d \).

**Identify:** This situation is analogous to having two capacitors \( C_1 \) in series, each with separation \( \frac{1}{4}(d-a) \).

**Set Up:** For capacitors in series,

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}. \]

**Execute:**

(a) \[ C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \frac{1}{2} C_1 = \frac{\varepsilon_0 A}{d-a/2} = \frac{\varepsilon_0 A}{d-a} \]

(b) \[ C = \frac{\varepsilon_0 A}{d-a} = \frac{\varepsilon_0 A}{d-a} = \frac{\varepsilon_0 A}{d-a} = \frac{C_0 d}{d-a} \]

(c) As \( a \to 0 \), \( C \to C_0 \). The metal slab has no effect if it is very thin. And as \( a \to d \), \( C \to \infty \). \( V = Q/C \). \( V = E_y \) is the potential difference between two points separated by a distance \( y \) parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large \( Q \) on the plates for a given potential difference. Since \( Q = CV \) this corresponds to a very large \( C \).
24.43. IDENTIFY: The permittivity $\varepsilon$ of a material is related to its dielectric constant by $\varepsilon = K\varepsilon_0$. The maximum voltage is related to the maximum possible electric field before dielectric breakdown by $V_{\text{max}} = E_{\text{max}} d$. $E = \frac{E_{\text{max}}}{K} = \frac{\sigma}{K\varepsilon_0}$, where $\sigma$ is the surface charge density on each plate. The induced surface charge density on the surface of the dielectric is given by $\sigma_i = \sigma (1 - 1/K)$.

SET UP: From Table 24.2, for polystyrene $K = 2.6$ and the dielectric strength (maximum allowed electric field) is $2 \times 10^7$ V/m.

EXECUTE: (a) $\varepsilon = K\varepsilon_0 = (2.6)\varepsilon_0 = 2.3 \times 10^{-11}$ C$^2$/N·m$^2$

(b) $V_{\text{max}} = E_{\text{max}} d = (2.0 \times 10^7$ V/m$) (2.0 \times 10^{-3}$ m$) = 4.0 \times 10^4$ V

(c) $E = \frac{\sigma}{K\varepsilon_0}$ and $\sigma = \varepsilon E = (2.3 \times 10^{-11}$ C$^2$/N·m$^2$) (2.0 $\times 10^7$ V/m) = $0.46 \times 10^{-3}$ C/m$^2$.

$\sigma_i = \sigma (1 - 1/K) = (0.46 \times 10^{-3}$ C/m$^2$) (1 - 1/2.6) = $2.8 \times 10^{-4}$ C/m$^2$.

EVALUATE: The net surface charge density is $\sigma_{\text{net}} = \sigma - \sigma_i = 1.8 \times 10^{-4}$ C/m$^2$ and the electric field between the plates is $E = \sigma_{\text{net}} / \varepsilon_0$.

24.47. A 12.5-μF capacitor is connected to a power supply that keeps a constant potential difference of 24.0 V across the plates. A piece of material having a dielectric constant of 3.75 is placed between the plates, completely filling the space between them. (a) How much energy is stored in the capacitor before and after the dielectric is inserted? (b) By how much did the energy change during the insertion? Did it increase or decrease?

24.47. IDENTIFY: $C = KC_0$, $U = \frac{1}{2}CV^2$.

SET UP: $C_0 = 12.5 \mu$F is the value of the capacitance without the dielectric present.

EXECUTE: (a) With the dielectric, $C = (3.75)(12.5 \mu$F) = 46.9 μF.

before: $U = \frac{1}{2}C_0V^2 = \frac{1}{2}(12.5 \times 10^{-6}$ F$)(24.0$ V$)^2 = 3.60$ mJ

after: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9 \times 10^{-6}$ F$)(24.0$ V$)^2 = 13.5$ mJ

(b) $\Delta U = 13.5$ mJ - 3.6 mJ = 9.9 mJ. The energy increased.

EVALUATE: The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.
25.14. A wire 6.50 m long with diameter of 2.05 mm has a resistance of 0.0290 Ω. What material is the wire most likely made of? 

**Evaluate:** A tangle electric item is required on an unishe, because it has a larger resistivity.

**Identify:** The resistivity of the wire should identify what the material is.

**Set up:** \( R = \rho l / A \) and the radius of the wire is half its diameter.

**Execute:** Solve for \( \rho \) and substitute the numerical values.

\[
\rho = \frac{AR}{L} = \frac{\pi (D/2)^2}{6.50 \text{ m}} \left( \frac{0.0205 \text{ m}}{2} \right)^2 (0.0290 \text{ Ω}) = 1.47 \times 10^{-4} \text{ Ω} \cdot \text{m}
\]

25.58. A plastic tube 25.0 m long and 4.00 cm in diameter is dipped into a silver solution, depositing a layer of silver 0.100 mm thick uniformly over the outer surface of the tube. If this coated tube is then connected across a 12.0-V battery, what will be the current? 

**Identify:** Use \( R = \frac{\rho l}{A} \) to calculate the resistance of the silver tube. Then \( I = V / R \).

**Set up:** For silver, \( \rho = 1.47 \times 10^{-8} \text{ Ω} \cdot \text{m} \). The silver tube is sketched in Figure 25.58. Since the thickness \( T = 0.100 \text{ mm} \) is much smaller than the radius, \( r = 2.00 \text{ cm} \), the cross section area of the silver is \( 2\pi r T \). The length of the tube is \( l = 25.0 \text{ m} \).

**Execute:** \[
I = \frac{V}{R} = \frac{V}{\rho l / A} = \frac{V}{\rho l} = \frac{V(2\pi T)}{(2\pi)(20.0 \times 10^{-5} \text{ m})(0.100 \times 10^{-3} \text{ m})} = \frac{12 \text{ V}(2\pi)(2.00 \times 10^{-5} \text{ m})(0.100 \times 10^{-3} \text{ m})}{(1.47 \times 10^{-8} \text{ Ω} \cdot \text{m})(25.0 \text{ m})} = 410 \text{ A}
\]

**Evaluate:** The resistance is small, \( R = 0.0292 \text{ Ω} \), so 12.0 V produces a large current.

![Figure 25.58](image)

26.2. A machine part has a resistor X protruding from an opening in the side. This resistor is connected to three other resistors, as shown in Fig. 26.38. An ohmmeter connected across a and b reads 2.00 Ω. What is the resistance of X?

**Identify:** It may appear that the meter measures X directly. But note that X is in parallel with three other resistors, so the meter measures the equivalent parallel resistance between ab.

**Set up:** We use the formula for resistors in parallel.

**Execute:** \[
1/(2.00 \Omega) = 1/X + 1/(15.0 \Omega) + 1/(5.0 \Omega) + 1/(10.0 \Omega)
\]

**Evaluate:** \( X = 7.5 \Omega \).