

Physics 3053 : Methods of Mathematical Physics in Physical Sciences . Fall 2006:

Scope of the course and pre-requisites:

3053 (3 hours)

Methods of Mathematical Physics in Physical Sciences

Broad introduction to analytical techniques used in upper-level physics courses. Various approaches to problems in optics and waves, electromagnetism, quantum theory, thermodynamics, and statistical mechanics will be covered. Prerequisites: Phys 2073, Math 3073.

Course Time: Tu, Th : 2-3.15 p.m, Room M4 KEH

Instructor: Dr. Parameswar Hari

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**Text: Mathematical methods in the physical sciences by M. L. Boas, Third edition
Published by John Wiley (2006)**

Additional texts for reference:

- 1) **Classical Mechanics by H. Goldstein, Second edition, Addison Wesley (1980)**
- 2) **Introduction to Quantum Mechanics: D. Griffiths, Second Edition, Pearson (2004)**
- 3) **Introduction to Electrodynamics, by D. Griffiths, Second Edition, Prentice Hall (1989)**

Office hours: W: 10 a.m. -12 p.m. Additional hours available by appointment

Description of the course

This course is aimed at developing the necessary mathematical techniques required in junior and senior level physics and engineering physics courses. The main emphasis of this course will be on problem solving rather than on proofs and justifications of techniques. In addition, this course will not require any computer based problem solving. Students are required to solve problems and graph solutions (sketch the approximate

nature of solutions for physical understanding) without using computer or even a calculator.

In this course, students will be introduced to the applications of mathematical techniques in electricity, magnetism, mechanics, quantum mechanics, solid state physics, optics, thermal physics and statistical mechanics as described below:

Part 1 : Mathematics of Electromagnetism and Waves (Aug. 22 – Sept. 14)

Chapter 2, 6, and 7

Part 2: Mathematics of Classical mechanics (Sept19 – Oct.5)

Chapter 5 and 9 (Oct. 10- Oct. 31)

Part 3: Mathematics of Quantum Mechanics (Nov. 2- Nov. 30)

Chapter 3, 4, 8, 11 and 12

1. LCR Circuits and Damped Harmonic Oscillators

Complex number representation of the LCR circuit. Phase, amplitude and impedance of the LCR circuit. Various solution to the driven harmonic oscillator using complex functions (overdamped, critically damped and resonance). Representation of Complex functions in two dimensions. Phasor diagrams of AC circuits.

2. Hilbert Space and Operator Algebra

Applications to simple rotations in two dimensions. Vectors in two dimensions. Addition of vectors. Invariance of vectors under rotation. The Lorentz transformation in relativity. The Hilbert space in quantum mechanics. Linear operators and eigen vectors. The eigen value problem. Coupled oscillators. Multiplication of matrices and non-commutation. Non-commuting operators in quantum mechanics. Operators represented as matrices. Linear combinations, linear functions and orthogonality. Applications to quantum mechanics.

3. Solutions to the Wave Equation

Maxwell's relations in thermodynamics. Change of variables. The wave equation and solution to wave equation Differentiation of integrals. The heat conduction equations and Fick's law. Method of Lagrangian multipliers in mechanics. Equations of motion. Simple harmonic motion.

4. Calculations of Moments of Inertia and Volume Elements

Area of a rectangular plate. Calculation of moment of inertia and center of gravity. Change of variables. The Jacobian. Spherical, cylindrical and Cartesian system of coordinates. Transformation from one coordinate system to another using the Jacobian.

5. Vector Analysis Applied to Electrodynamics

Physical meaning of divergence, gradient and curl. Examples of gradient and curl from electricity and magnetism. Stokes theorem and Green's theorem. Maxwell's equations represented in terms of the curl and divergence of electric and magnetic field vectors.

6. Simple Harmonic Motion and Periodic Motion

Simple harmonic motion and wave motion. Periodic functions. Applications of Fourier series to square, saw tooth and rectified half-wave functions. Sound waves. Complex Fourier series. Integration of even and odd functions from symmetry. Parseval's theorem. Plane wave representation. Fourier transforms. Examples from solid state physics. The Bloch theorem.

7. Harmonic Oscillator and Resonance

Simple LRC circuits. Separation of variables. Charging a capacitor. R-L circuit. First order differential equation. Radioactive decay. Bernoulli's equation. Homogeneous equations. The auxiliary equations. Damped harmonic oscillator. Forced oscillations. Method of undetermined coefficients. Use of Fourier series in finding a particular solution. Laplace transformation. LCR circuit solution from Laplace transforms. The Dirac delta function and the step function. A point charge potential as a delta function. Introduction to Green's functions. Examples from electrostatics.

8. The Principle of Least Action

The Euler equations. The Brachistochrone problem. Lagrange's equations for a particle moving on the surface of a cone. Coupled pendulum. Isoperimetric problems.

9. Particle Motion in Various Potentials

Gamma function. Recursion relation. Solution of a particle moving in special potentials. Special formulas involving Gamma function. Evaluation of integrals. The beta function. Period of a simple pendulum. The error function. The Stirling's formula. Application to statistical mechanics. Elliptic functions. Pendulum with large amplitudes.

10. Angular Momentum and Scattering from a Spherical Potential

Introduction to the applications Bessel, Legendre, Laguerre and Hermite polynomials. Examples from quantum mechanics: the hydrogen atom, the harmonic oscillator, inverse square law potential and scattering from a spherical potential. Generating functions. Orthogonal functions. Spherical harmonics. Angular momentum. Ladder operators. Solution to harmonic oscillator in terms of ladder operators.

Grading

Your final course grade will be determined by the following items and weights:

| <u>Item</u> | <u>% of Final Course Grade</u> |
|--------------------------------|--------------------------------|
| Homework | 50.0% |
| Two Exams | 20.0% |
| Final Exam (cannot be dropped) | 30.0% Comprehensive |
| <hr/> | |
| Total | 100.0% |

I will strictly adhere to the following criteria in assigning your final course grade:

| <u>Final Course %</u> | <u>Final Course Grade</u> | <u>Meaning</u> |
|-------------------------|---------------------------|----------------|
| $89.5 \leq \% \leq 100$ | A | Superior |
| $79.5 \leq \% < 89.5$ | B | Good |
| $69.5 \leq \% < 79.5$ | C | Average |
| $59.5 \leq \% < 69.5$ | D | Poor |
| $0 \leq \% < 59.5$ | F | Failure |

Homework:

Homework is an extremely important component of this course. You learn the various methods of mathematical physics by working out large number of problems. We will discuss homework assignments on every Tuesday during the first 15 minutes of the class hour. Homework carries significant weight in this course (50%). I strongly encourage you to consult with me during my office hours, if you are stuck with a homework problem. Homework assignments consist of problems for each chapter we cover in the text. I will provide detailed solutions to each homework assignment shortly after it is due.

Two exams will be given during the semester. Final exam will be comprehensive.

If you withdraw from this course prior to the start of the fourth week of the semester, the course will not be shown on your academic record. If you withdraw from this course after the start of the fourth week and up to and including the twelfth week of the

semester, you will receive a grade of W (withdraw) or WF (withdraw failing) depending on your grade in the course at the time of your withdrawal.

If you are doing passing work but are unable to complete your course work due to a legitimate and documented extenuating circumstance (serious illness or personal problems, for instance), at my discretion I may grant you a grade of "I" (incomplete). I will only grant an incomplete for an exceptionally good reason. To receive an incomplete, you will need to sign a "Record of Incomplete" form that will specify what work you must do and when the work must be finished to remove the incomplete. This form will be filed in the dean's office. The incomplete grade can remain on your record for one year. If the unfinished work is not completed in that time, your course grade will change from an I to an F.

Academic Misconduct

*Cheating is usually of little value and is surprisingly easy to detect. I have no tolerance for cheating and will pursue it aggressively. I will follow the "Policies and Procedures Relating to Student Academic Misconduct in the College of Engineering and Applied Sciences" which may be obtained from the Dean's office. The *minimum* penalty for cheating is a zero grade on the assignment in question that cannot then be dropped in the calculation of the final grade.*

The work you submit for grading must be your own and must demonstrate some level of original thinking. Verbatim copying or even edited copying of assignments from other sources that demonstrates no original thinking constitutes cheating.